3 PHASE AC MOTOR STARTING METHODS:

A Detailed Analysis of Various Methods of Starting an AC Induction Motor and Their Effects on Motor Characteristics.

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This paper will explain basic AC induction motor theory and then apply this theory to describe motor characteristics during starting. Then this paper discusses the effects soft starting an AC motor has on these various characteristics. Lastly, this paper briefly reviews various methods of soft starting an AC motor and discusses some of the advantages and disadvantages of each of these methods.

II. AC MOTOR CIRCUIT THEORY:

The standard 3 phase AC induction motor has two main components called the rotor and stator. The stator is constructed of a laminated steel core with an interwoven winding. The stator winding is brought out from the motor usually to a terminal or junction box mounted to the motor stator for electrical connection to the motor. The rotor is a laminated core mounted inside the stator housing and is coupled to the shaft of the motor. The stator winding circuit is designed in groups of poles. Each phase of a pole group is electrically separated by 120 degrees. If a voltage is applied to the stator winding at some given frequency, then a field is generated internal of the stator that causes the rotor to want to run at a rotational speed defined by the equation below:

$$\text{Synchronous\_RPM} := \frac{120 \cdot \text{Frequency}}{\text{Number\_of\_Poles}}$$

For a 60 Hz system, the synchronous speed of various quantities of motor poles is shown below.

Poles_I	Synchronous_RPM_I
2	3600
4	1800
6	1200
8	900
10	720
12	600
14	514.3
16	450
18	400
20	360

TABLE #1: Synchronous Speeds of 60Hz Motors.

For a 50 Hz system, the synchronous speed of various quantities of motor poles is shown below.

Poles_I	Synchronous_RPM_I
2	3000
4	1500
6	1000
8	750
10	600
12	500
14	428.6
16	375
18	333.3
20	300

TABLE #2: Synchronous Speeds of 50Hz Motors.

The above listed values are values of synchronous speeds. This is when the rotor is in synchronous speed with the rotating field inside the stator. However, the way a standard AC motor develops torque is by allowing the rotor to lag behind the rotating field inside the stator. This is called slip. The value of slip is defined as the ratio of the difference between rotor synchronous speed and rotor actual speed as compared to the rotor synchronous speed. Mathematically, this is defined as:

$$Slip := \frac{Synchronous\_RPM - Full\_Load\_RPM}{Synchronous\_RPM}$$

The table below defines various amounts of rated slip for various pole count machines and their respective rated (or full load) speeds.

% Slip	60Hz Motors									
	2 Pole	4 Pole	6 Pole	8 Pole	10 Pole	12 Pole	14 Pole	16 Pole	18 Pole	20 Pole
0	3600	1800	1200	900	720	600	514	450	400	360
1	3564	1782	1188	891	713	594	509	446	396	356
2	3528	1764	1176	882	706	588	504	441	392	353
3	3492	1746	1164	873	698	582	499	437	388	349
4	3456	1728	1152	864	691	576	494	432	384	346
5	3420	1710	1140	855	684	570	489	428	380	342
6	3384	1692	1128	846	677	564	483	423	376	338
7	3348	1674	1116	837	670	558	478	419	372	335
8	3312	1656	1104	828	662	552	473	414	368	331
9	3276	1638	1092	819	655	546	468	410	364	328
10	3240	1620	1080	810	648	540	463	405	360	324
11	3204	1602	1068	801	641	534	458	401	356	320
12	3168	1584	1056	792	634	528	453	396	352	317
13	3132	1566	1044	783	626	522	447	392	348	313
14	3096	1548	1032	774	619	516	442	387	344	310
15	3060	1530	1020	765	612	510	437	383	340	306
16	3024	1512	1008	756	605	504	432	378	336	302
17	2988	1494	996	747	598	498	427	374	332	299
18	2952	1476	984	738	590	492	422	369	328	295
19	2916	1458	972	729	583	486	417	365	324	292
20	2880	1440	960	720	576	480	411	360	320	288
21	2844	1422	948	711	569	474	406	356	316	284
22	2808	1404	936	702	562	468	401	351	312	281
23	2772	1386	924	693	554	462	396	347	308	277
24	2736	1368	912	684	547	456	391	342	304	274
25	2700	1350	900	675	540	450	386	338	300	270

TABLE #3: Rated Slip Speeds of 60Hz Motors.

% Slip	50Hz Motors									
	2 Pole	4 Pole	6 Pole	8 Pole	10 Pole	12 Pole	14 Pole	16 Pole	18 Pole	20 Pole
0	3000	1800	1200	900	720	600	514	450	400	360
1	2970	1485	990	743	594	495	424	371	330	297
2	2940	1470	980	735	588	490	420	368	327	294
3	2910	1455	970	728	582	485	416	364	323	291
4	2880	1440	960	720	576	480	411	360	320	288
5	2850	1425	950	713	570	475	407	356	317	285
6	2820	1410	940	705	564	470	403	353	313	282
7	2790	1395	930	698	558	465	399	349	310	279
8	2760	1380	920	690	552	460	394	345	307	276
9	2730	1365	910	683	546	455	390	341	303	273
10	2700	1350	900	675	540	450	386	338	300	270
11	2670	1335	890	668	534	445	381	334	297	267
12	2640	1320	880	660	528	440	377	330	293	264
13	2610	1305	870	653	522	435	373	326	290	261
14	2580	1290	860	645	516	430	369	323	287	258
15	2550	1275	850	638	510	425	364	319	283	255
16	2520	1260	840	630	504	420	360	315	280	252
17	2490	1245	830	623	498	415	356	311	277	249
18	2460	1230	820	615	492	410	351	308	273	246
19	2430	1215	810	608	486	405	347	304	270	243
20	2400	1200	800	600	480	400	343	300	267	240
21	2370	1185	790	593	474	395	339	296	263	237
22	2340	1170	780	585	468	390	334	293	260	234
23	2310	1155	770	578	462	385	330	289	257	231
24	2280	1140	760	570	456	380	326	285	253	228
25	2250	1125	750	563	450	375	321	281	250	225

TABLE #4: Rated Slip Speeds of 50Hz Motors.

The amount of slip required to develop a certain amount of available torque at the motor shaft is very much dependant on the construction of the motor. NEMA (National Electronics Manufacturing Association) has classified motors with respect to their torque vs. speed characteristics. NEMA B design motors are low starting torque motors. That means that the available locked rotor torque value of the motor is less than other types of motor designs. These motors are used on variable torque applications where the starting torque requirements are relatively low. These motors are one of the most common types of motors on the market today. NEMA C design motors have a higher starting torque value. That means that the available locked rotor torque value of the motor is higher than NEMA B design motors. These motors are used where the load requires a large value of torque to start rotation. NEMA D design motors have large values of rated slip. Where other NEMA designs have rated slip values between 1 to 5%, the NEMA D design rated slip values range between 5 to 15% depending on construction. This means that the motor's rotor slows much more at rated motor torque than other NEMA designs. These motors are used in high peak torque load requirements where system inertia is insufficient to provide the momentary energy to keep a motor at speed. NEMA E design motors are energy efficient motors. One of the more important differences of NEMA E design motors and other motors is the magnitude of the locked rotor current of the motor. Most other NEMA design motors have a value of locked rotor current that is about 6 times motor full load current. NEMA E design motors have a value of locked rotor current that is normally 8 times full load current. The figure below provides an indication of these various NEMA motor designs and their torque characteristics as the motor is accelerated to full speed.

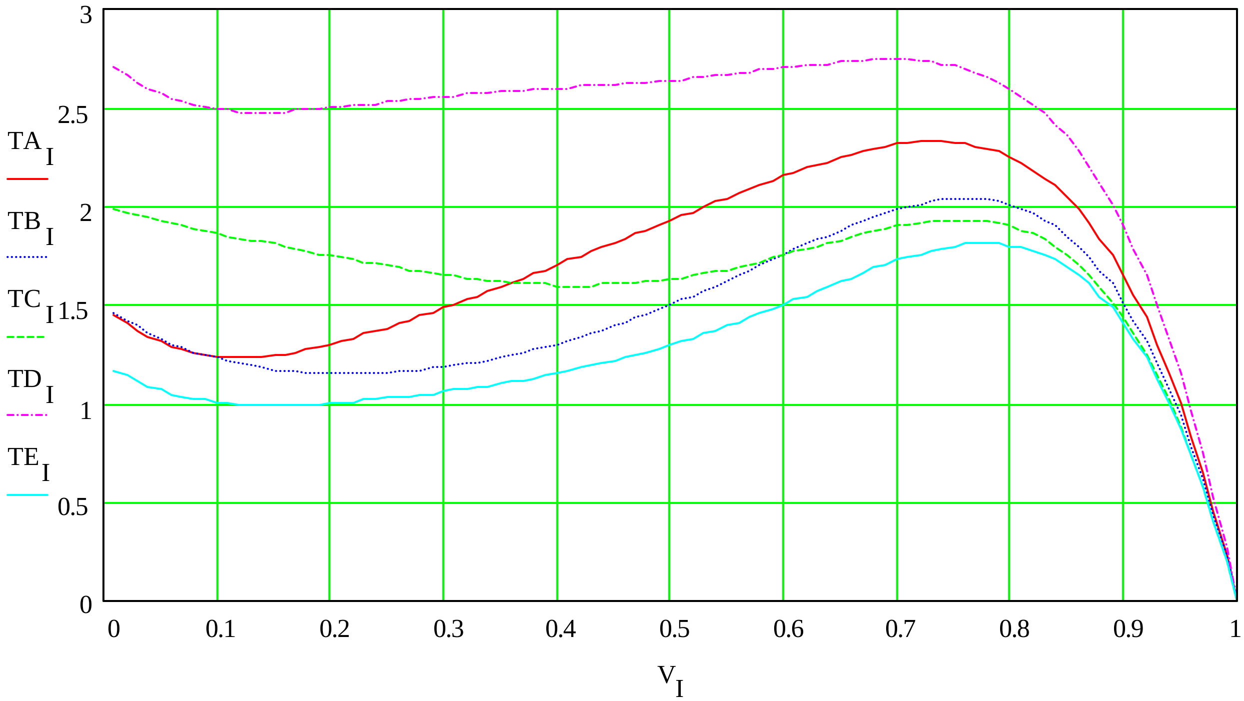

FIGURE #1: Graph of Torque Vs. Speed for Various NEMA Design Motors.

Where:
V = Velocity of the motor rotor in percent of motor synchronous speed.
TA = NEMA A design motor available torque.
TB = NEMA B design motor available torque.
TC = NEMA C design motor available torque.
TD = NEMA D design motor available torque.
TE = NEMA E design motor available torque.

As Figure #1 shows, the motor available torque is dependant on motor rotor speed and motor construction. Actually, motor available torque is dependent on motor stator voltage also as will be discussed when we get to the soft start section of this paper. Now that we understand motor speed or slip and how it affects motor torque, we can define motor power with respect to motor speed. The amount of power available at the shaft of the motor is the product of the motor available torque at the shaft of the motor and the rotational speed of the shaft. This is mathematically defined below as:

$$\text{Horsepower} := \frac{\text{Torque} \cdot \text{RPM}}{5250}$$

We can derive the horsepower vs. speed curve by taking the value of torque in the torque vs. speed curve and multiplying it by the speed at each point in the graph to find available power from the motor as the speed of the rotor is varied. This can be seen graphically in Figure 2 below:

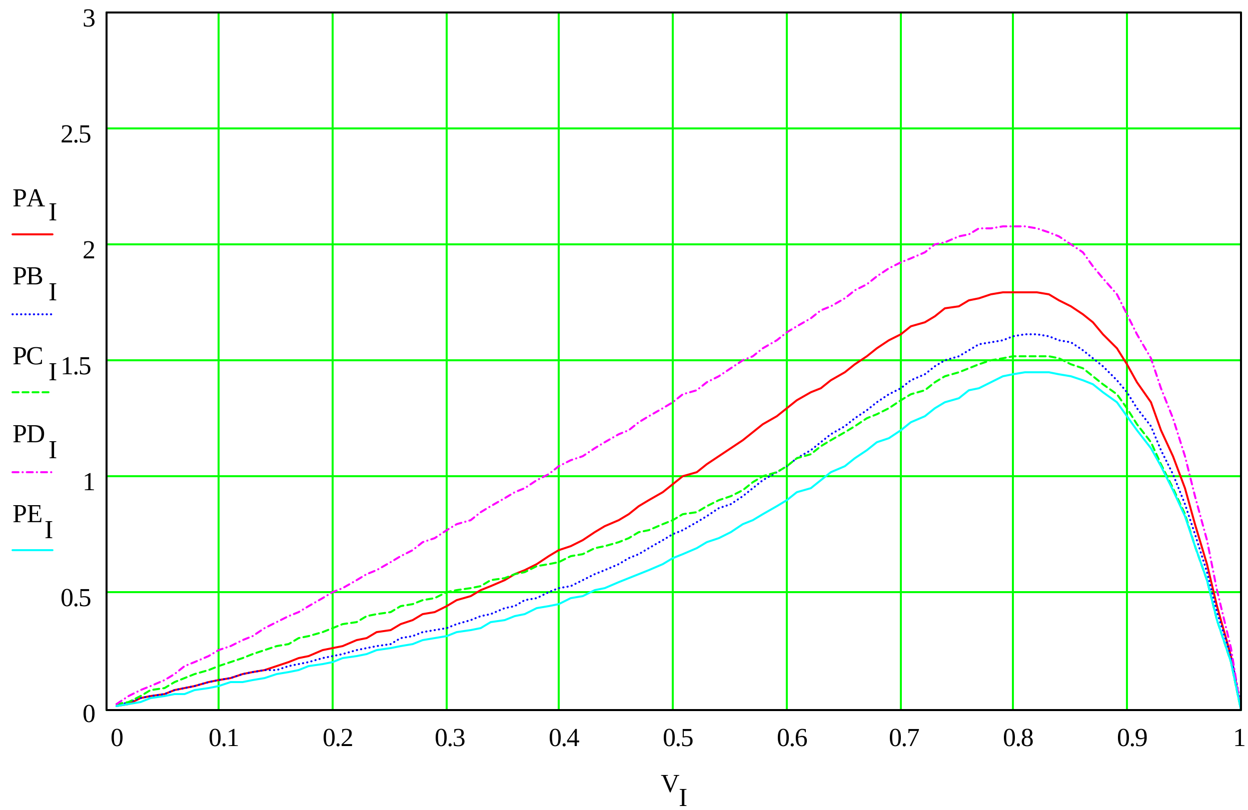

FIGURE #2: Graph of Horsepower Vs. Speed for Various NEMA Design Motors..

Where:
V = Velocity of the motor rotor in percent of motor synchronous speed.
PA = NEMA A design motor available horsepower.
PB = NEMA B design motor available horsepower.
PC = NEMA C design motor available horsepower.
PD = NEMA D design motor available horsepower.
PE = NEMA E design motor available horsepower.

As Figure #2 shows, the motor available power is dependant on motor rotor speed and motor construction just like motor torque. Actually, motor available power is dependent on motor stator voltage also since it is a function of motor torque and will be discussed when we get to the soft start section of this paper.

The next important issue is shaft mechanical stress. The mechanical stress on the shaft of a motor is proportional to the power transmitted through the shaft and is inversely proportional to the rotational speed of the shaft and the cube of the diameter of the shaft. This is described mathematically as:

$$\text{Shaft\_Stress} := \frac{HP \cdot 321000}{RPM \cdot D^3}$$

Since we already know that HP is the product of torque and speed, we can rewrite the above equation in terms of shaft torque as shown below:

$$Shaft\_Stress := \frac{Torque \cdot 321000}{5250 \cdot D^3}$$

As is shown in the above equation, shaft stress is directly proportional to transmitted torque and inversely proportional to the cube of the diameter of the shaft. This equation will come to play later when we discuss the capabilities of reduced voltage starting and how it effects shaft stress by limiting available motor torque.

Now that we understand motor available torque and power as it accelerates from zero speed to full speed, we can turn our attention to the time it takes a motor to reach full speed. A very important concept to grasp in understanding accelerating time is to understand that load inertia and load torque requirement are two separate mechanical issues. Load inertia is a measure of the amount and configuration of mass a system contains. The larger the amount of mass in a system, the larger the amount of energy that is required to accelerate that mass to speed. In a pure system, (one with no frictional losses), once this mass is running at a certain speed, no energy is required to maintain this speed. In a real system, (one with frictional losses), once this mass is running at a certain speed, the only energy required is that amount needed to overcome the frictional losses to keep the mass at this set speed. Load torque requirement on the other hand is a measure of the torque that the mechanical load requires at a certain speed to develop the power required by the load. A good example to explain the difference is a mechanical system with a fly wheel and a clutch. With the clutch dis-engaged, the only energy required from the motor to accelerate the flywheel is the magnitude of energy to accelerate the inertia up to speed. Once the inertia is at speed, since the clutch is dis-engages, the motor will deliver minimal power, since no work is being performed by the mechanical system. Once the clutch is engaged, and the final load is added to the system, now the motor must provide torque and, therefore, power to the load to develop the power required by the final load. The time a motor takes to reach full speed is directly proportional to the magnitude of the system inertia (as reflected to the shaft of the motor) and proportional to the magnitude of the required speed change (usually from zero speed to full speed if the motor is not wind-milling), and inversely proportional to the magnitude of the average accelerating torque of the motor. This last value of average accelerating torque of a motor in a system is probably the most mis-understood value in this subject. The accelerating time can be mathematically shown by:

$$Accel\_Time := \frac{WK \cdot DeltaRPM}{308 \cdot AAT}$$

WK is the inertial of the motor plus the inertia of the system as reflected to the shaft of the motor. When a transmission contains gear ratios, the inertial reflects through the gear box by the square of the ratio of the gearing. This is mathematically shown below as:

$$WK := Rotor\_Inertia + Load\_Inertia \cdot \left(\frac{Load\_RPM}{Motor\_RPM}\right)^2$$

The last parameter that effects the time for a motor to accelerate is the average accelerating torque. This value is the value of the motor available torque minus any load torque requirement at the motor as it accelerates to full speed. If we take a look at a torque vs. speed curve for a standard NEMA B design motor shown below in figure #3, we see that the area under the torque speed curve is the value of the motor average accelerating torque. This is the amount of torque available from the motor from zero to full speed averaged over the entire speed range. In figure #3 below, this value is the magnitude of the area under the red curve or the area in yellow.

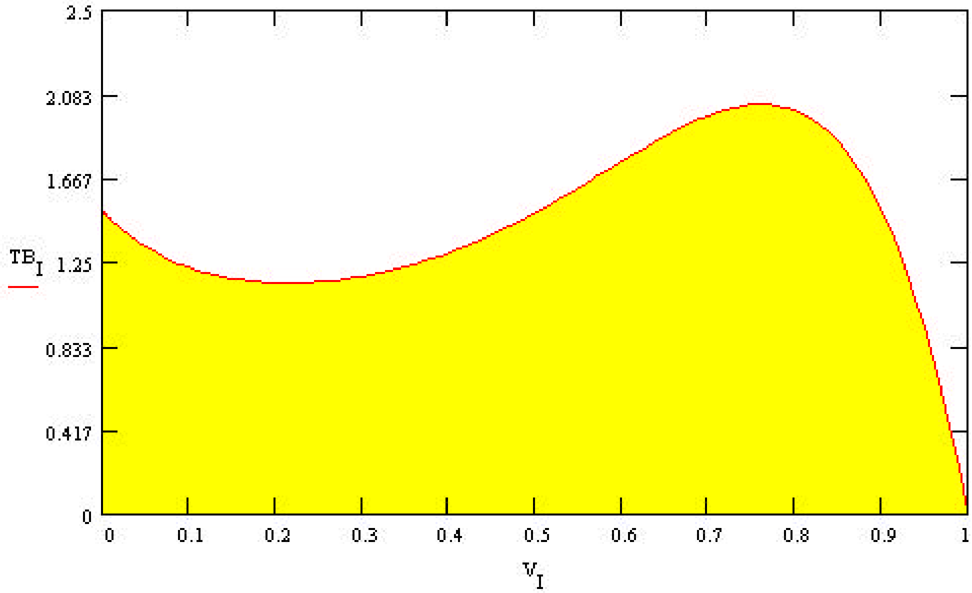

FIGURE #3: Graph of Horsepower Vs. Speed for Various NEMA Design Motors

For the example in Figure #3, the value of motor average accelerating torque is found to be 144.5% of full load from the formula:

$$AMAAT := \frac{1}{K1} \cdot \sum_{I=1}^{K1} TB_I \qquad AMAAT = 1.445$$

Another commonly used formula for calculating the value of this area under the torque curve is shown below. Note from the results of the example that this is only an approximation and does provide error.

$$\text{Avg Acc Trq} = \frac{[(FLT + BDT)/2] + BDT + LRT}{3}$$

Using this equation and values of 100% for FLT, 203.6% for BDT, and 146% for LRT as derived from Figure #3, the above equation gives us a value for motor average accelerating torque to be 167.1% of motor full load torque.

$FLT := 1 \qquad BDT := 2.036 \qquad LRT := 1.46$

$$EMAAT := \frac{\left(\frac{FLT + BDT}{2}\right) + BDT + LRT}{3} \qquad EMAAT = 1.671$$

In the above equation, FLT is the full load torque and is defined as the value of torque that a motor will develop when the rotor is running at rated RPM. BDT is the value of break down torque and is defined as the value of torque that a motor develops at the maximum of the torque curve. This is the point at which either an increase in motor speed or a decrease in motor speed will develop less torque from the motor and is about 80% speed for the curve in Figure #3. LRT is the locked rotor torque and is the value of motor available torque when full voltage is applied to the stator circuit and the motor rotor is at a stand still (i.e. locked). As can be seen from our results, while the formula above does provide a good approximation to the motor available torque, in most circumstances this is not the value of average accelerating torque. This is only a reasonable approximation for average accelerating torque when the load torque requirement of the mechanical load is zero. When load toque requirement is zero, since average accelerating toque is the motor available torque minus the load required torque, the motor available torque is the value of average accelerating torque. However, in most applications when the motor is accelerated from zero to full speed, the load is performing some mechanical work which requires some amount of torque. Since this load torque requirement is the amount of torque the load requires to maintain a certain speed, the value of torque available from the motor to accelerate the load is the value of motor accelerating torque MINUS the value of load required torque. Another issue that makes this calculation even more difficult is that fact that the load torque speed curve varies depending on application just line motor torque curves vary depending on motor construction. In general, applications can be broken into two basic types, constant torque and variable torque. Constant torque applications such as conveyors require the approximately the same amount of torque throughout the speed range as they do at full speed. Therefore, when a motor is ½ way to speed, the load torque demand is the same as it is at full speed (approximately). Variable torque applications such as blowers require a reduced amount of torque as the speed is lowered from full speed. Centrifugal affinity laws dictate that the power requirement of a variable torque load is proportional to the cube of speed. Since torque is the power of the application divided by the speed of the application, it follows that the torque requirement of a variable torque load is proportional to the square of speed. Therefore, when a motor is ½ way to speed, the load torque demand is ¼ of the full speed value (approximately). Because of this characteristic, variable torque loads require much less load torque from the motor while accelerating and leave more torque available from the motor for acceleration. This is pictorially shown below in Figures #4 and #5.

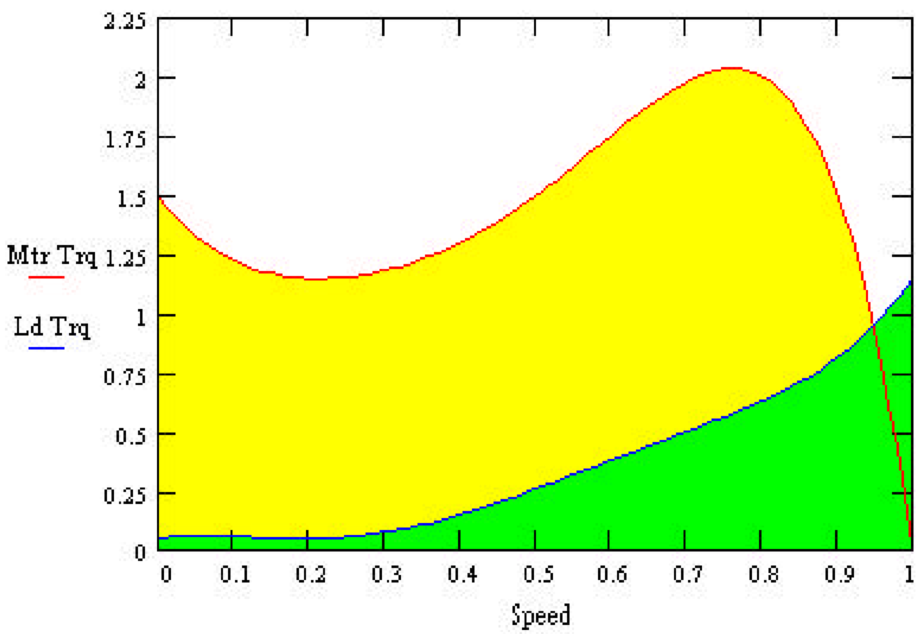

FIGURE #4: Graph of Horsepower Vs. Speed for NEMA B Design Motor Connected to a Variable Torque Load:

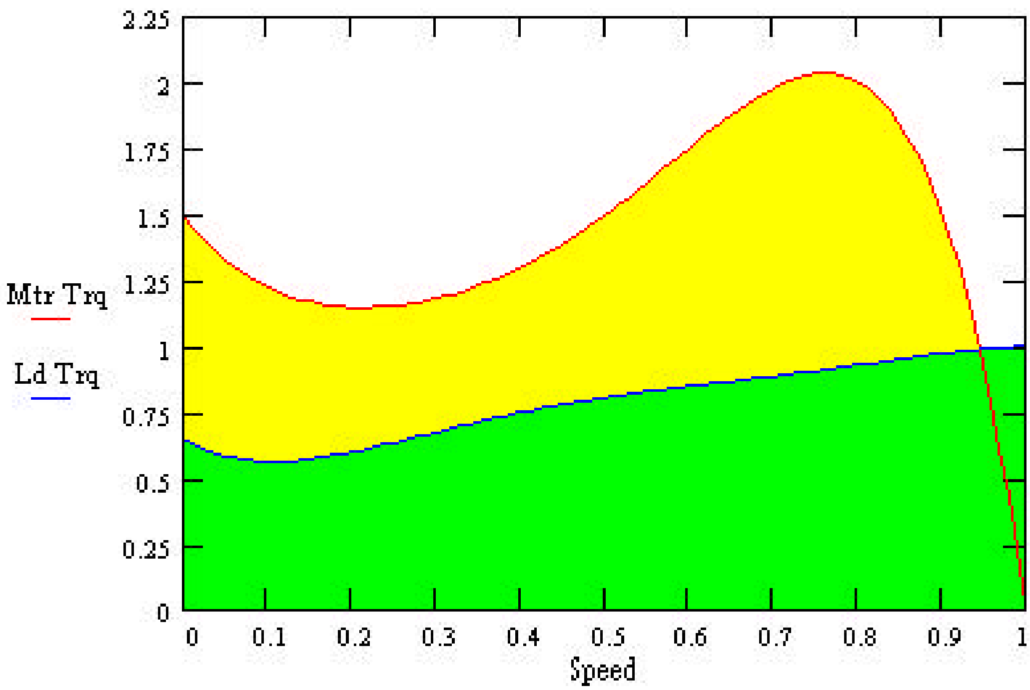

FIGURE #5: Graph of Horsepower Vs. Speed for NEMA B Design Motor Connected to a Constant Torque Load:

For the unloaded motor shown in Figure #3, we calculated the average accelerating torque to be 167.1% of motor full load torque. For the variable torque load curve of Figure #4, the value of average accelerating torque is now reduced to 109.1% of motor full load torque.

$$AMAAT := \frac{1}{K1} \cdot \left[\sum_{I=1}^{K1} TB_I - VTLT_I \right] \qquad AMAAT = 1.091$$

For the constant torque load curve of Figure #5, the value of average accelerating torque is now reduced to 65.8% of motor full load torque.

$$BMAAT := \frac{1}{K1} \cdot \left[\sum_{I=1}^{K1} TB_I - CTLT_I \right] \qquad BMAAT = 0.658$$

As is shown by Figures #3, #4 and #5, as the load torque requirement increases, the value of average accelerating torque is reduced accordingly.

Now that we understand general AC induction motor theory, we can now turn our attention to the question of how soft starting an AC induction motor effects it's mechanical and electrical parameters.

II. EFFECT OF REDUCED VOLTAGE STARTING ON AN AC INDUCTION MOTOR:

To understand how reducing the voltage to the stator of a motor effects motor characteristics during start, we must first evaluate the equivalent motor circuit model. Figure #6 below shows the equivalent motor circuit:

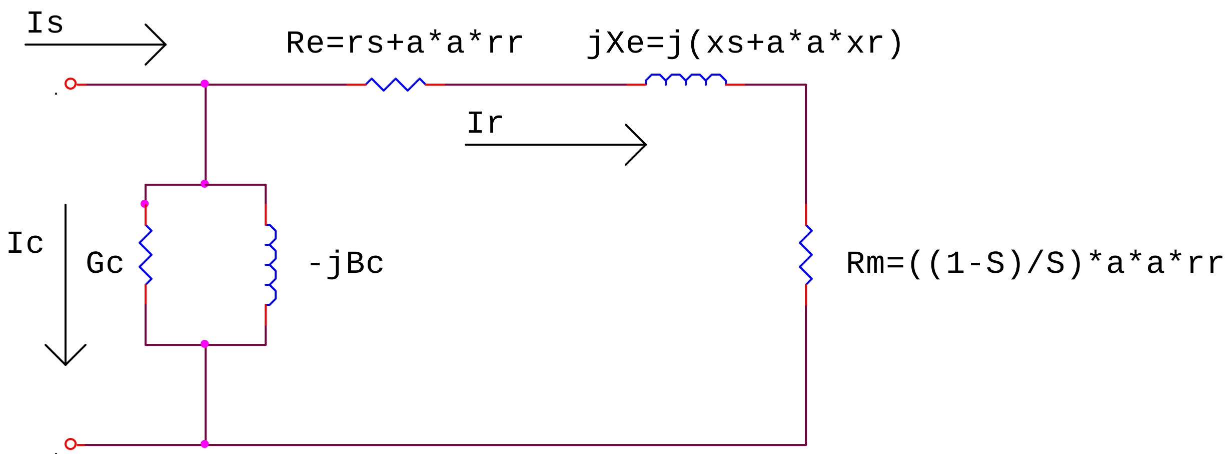

FIGURE #6: Equivalent Single Line Equivalent Circuit Model of Standard AC Induction Motor.

Where:
Rs = Resistance of the Stator.
Rr = Resistance of the Rotor.
Xs = Reactance of the Stator.
Xr = Reactance of the Rotor.
Gc = Shunt Conductance.
Bc = Shunt Subseptance.
A = Ratio of Stator Current to Rotor Current.
Is = Stator Circuit Current.
Ic = Current due to Shunt Admittance.
Ir = Rotor circuit current.

Since the above equivalent circuit represents the line to neutral values of the 3-phase motor, each value derived from this model is per phase values. Looking at the above equivalent circuit, at any particular speed, the circuit has certain impedance. Using ohms law, it can be seen that, at any given motor rotor speed, as the voltage is reduced at the stator of the motor, the current is reduced by the same proportion. The power in the gap is given by:

$$Pg := Ir^2 \cdot Rm$$

But, since power is a function of torque and speed, the above equation can be rewritten as:

$$T := \frac{Pg}{\Omega r}$$

Since the relation ship between rotor speed and stator rotational speed is the slip of the motor, we can rewrite the above equation as:

$$T := \frac{Pg}{\Omega s \cdot (1 - S)}$$

Now substituting the motor electrical parameters into the above equation, we finally come up the below equation.

$$T := \frac{(|Van|)^2}{\Omega s} \cdot \left[\frac{\frac{a^2 \cdot Rr}{S}}{\left(Rs + \frac{a^2 \cdot Rr}{S}\right)^2 + Xe^2} \right]$$

This equation shows us that the available torque from the motor is proportional to the SQUARE of the applied voltage. For example, if ½ of the nominal line voltage is applied to the motor stator, then the available torque is ¼ of the full voltage value.

Lastly, since power is proportional to torque and speed, at any given speed, the reduction in motor power will be the SQUARE of the reduction of applied voltage. Total power draw is 1.7 * V * I, and since I varies in proportion to V, we can deduce that the total power varies as the SQUARE of the reduction in stator voltage just like power. Since the power factor of a motor is the ratio of real power to total power and since real and total power both vary as the SQUARE of the applied voltage, we can state that the power factor of an AC induction motor is unaffected by the method of starting. EASA has a chart (shown in Figure #7) that shows the effects of various methods of reduced voltage starting on an AC induction motors locked rotor conditions.

<u>EASA Electrical Engineering Handbook</u>

STARTING CHARACTERISTICS OF
SQUIRREL CAGE INDUCTION MOTORS

STARTING METHOD	VOLTAGE AT MOTOR	LINE CURRENT	MOTOR TORQUE
Full-Voltage Value	100	100	100
Autotransformer			
80% tap	80	64*	64
65% tap	65	42*	42
50% tap	50	25*	25
Primary Resistor Typical Rating	80	80	64
Primary Reactor			
80% tap	80	80	64
65% tap	65	65	42
50% tap	50	50	25
Series-Parallel	100	25	25
Wye-Delta	100	33	33
Part-Winding (½- ½)			
2 to 12 Poles	100	70	50
14 and more Poles	100	50	50

FIGURE #7: Starting Characteristics of Squirrel Cage Induction Motors:

As we derived above, the amount of torque delivered by the motor is a function of the voltage applied to the stator windings of the motor. To be exact, the amount of motor available torque is reduced by the square of the reduction of voltage at the motor terminals.

$$T := \frac{(|Van|)^2}{\Omega s} \cdot \left[\frac{\frac{a^2 \cdot Rr}{S}}{\left(Rs + \frac{a^2 \cdot Rr}{S}\right)^2 + Xe^2} \right]$$

Since we know that HP is the product of torque and speed, we can derive that the available HP of the motor during start is also reduced by the square of the reduction of voltage at the motor terminals.

$$Horsepower := \frac{Torque \cdot RPM}{5250}$$

Also, since we know that shaft stress is proportional to motor torque and inversely proportional to the shaft diameter cubed, we can derive that the shaft stress is reduced by the square of the reduction of voltage at the motor terminals.

$$Shaft\_Stress := \frac{Torque \cdot 321000}{5250 \cdot D^3}$$

Average accelerating torque is one of the mo re misunderstood parameters and will be discussed in detail here. If we were to look at the common approximation for average accelerating torque shown below, we would conclude that, since the average accelerating torque is proportional to LRT, BDT, and FLT, as these values are reduced the value of average accelerating torque would be reduced accordingly.

$$\text{Avg Acc Trq} = \frac{[(FLT + BDT)/2] + BDT + LRT}{3}$$

However, this is only true for loads with not torque demand during acceleration. As we mentioned above, if there is no load tied to the motor during acceleration, then the entire amount of motor available torque is available to the system for acceleration and motor available torque equals the average accelerating torque. In this instance, since motor available torque is proportional to the square of the reduction of voltage, the value of average accelerating torque would also vary in proportion to the square of the reduction of voltage. However, most applications have a load tied to the motor during the acceleration period.   In these applications, the value of accelerating torque is the motor available torque minus the load required torque. Since the load required torque does not change with the method of starting the motor, when applying a soft start to a loaded motor for starting, the reduction of average accelerating torque is greater than the reduction of motor available torque.

This concept can best be described graphically by the example shown below. In Figure #4, we derived the average accelerating torque for a NEMA B design motor with a variable torque load started across the line and found it to be 109.1% of full load torque. In Figure #5, we derived the average accelerating torque for a NEMA B design motor with a constant torque load started across the line and found it to be 65.8% of full load torque. In Figure #8 below we find the torque speed curve for a NEMA B design motor with a variable torque load started with a soft start with a current limit set to 3 times FLA.

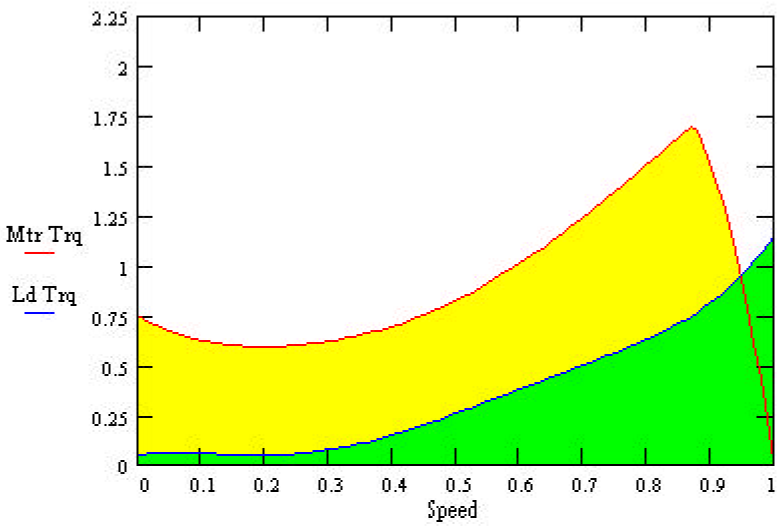

FIGURE #8: Soft Starting Characteristics of NEMA B Design Motors with Variable Torque Load:

The area under the red curve is the motor available torque and the area under the blue curve is the load-required torque. The area between the red curve and blue curve (or the area in yellow) is the value of average accelerating torque. The calculated average accelerating torque is found to be 56.2% of motor full load torque for this example as shown below.

$$CMAAT := \frac{1}{K1} \cdot \left[\sum_{I=1}^{K1} TBN_I - VTLT_I \right] \qquad CMAAT = 0.562$$

In Figure #9 below we find the torque speed curve for a NEMA B design motor with a constant torque load started with a soft start with a current limit set to 3 times FLA.

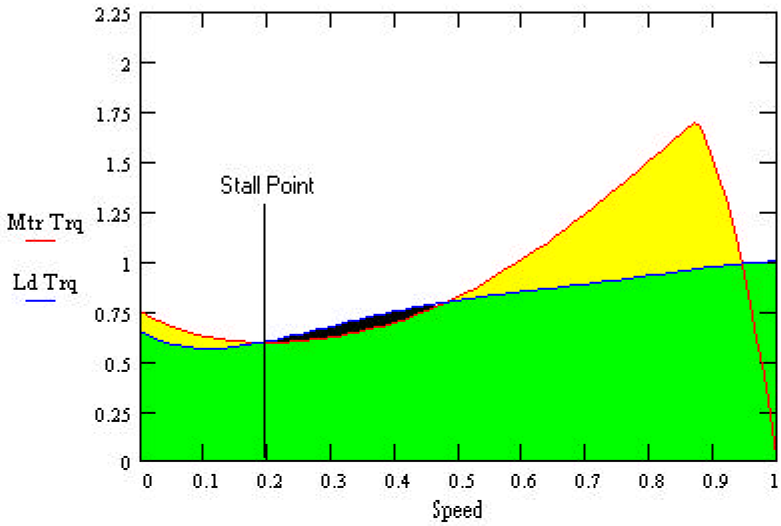

FIGURE #9: Soft starting Characteristics of NEMA B design Motors with Constant Torque Load:

The area under the red curve is the motor available torque and the area under the blue curve is the load-required torque. The area between the red curve and blue curve (or the area in yellow) is the value of average accelerating torque. The calculated average accelerating torque is found to be 12.9% of motor full load torque as shown below.

$$DMAAT := \frac{1}{K1} \cdot \left[\sum_{I=1}^{K1} TBN_I - CTLT_I \right] \qquad DMAAT = 0.129$$

For the variable torque load, the reduction of average accelerating torque was 56.2%/109.1% = 51.5%. For the constant torque load, the reduction of average accelerating torque was 12.9%/65.8% = 19.6%. So, as this example proves, the type of load torque requirement that exists while starting the motor greatly affects the value of average accelerating torque during a soft start cycle. This is true regardless of the method of soft starting. However, there is another very important issue that is shown by Figure #9. A closer look at Figure #9 above will reveal the fact that this motor / load / starting method will not allow the motor to accelerate to full speed. The motor would accelerate to 20% of full speed and then stall. Even though we have a positive value of average accelerating torque of 12.9% for this situation, the motor would be unable to accelerate to full speed. This is because between the speeds of 20% and 47% speed, the value of motor available torque is less than the load-required torque. Because of this, there is zero available accelerating torque between 20% and 47% speeds. This demonstrates a very important issue when applying soft start methods. Regardless of the method of soft starting a motor, the engineer in charge of the design must ensure that, at all points in the torque speed curve, the value of motor available torque is greater than the value of load required torque.

Next, we will evaluate the effect a soft start has on average accelerating time. First, let us look at the situation where the motor is only coupled to system inertia but has no load torque requirement. Since we know that acceleration time proportional to the inertia of the system, proportional to the change in speed of the motor, and inversely proportional to the average accelerating time, we can see that, if the average accelerating torque is reduced in half, then the value of accelerating time will double.

$$Accel\_Time := \frac{WK \cdot DeltaRPM}{308 \cdot AAT}$$

With applications where the motor has not load torque requirement associated during the start curve, we know that the value of average accelerating torque is proportional to the square of the reduction of voltage to the motor terminals. So for a motor that has ½ voltage applied to the stator, the value of average accelerating torque will be ¼ and the accel time will be 4 times that of an across the line start.

Now let us look at the more common situation where the motor has some load torque requirement associated with it during starting. Now the value of average accelerating torque is reduced by MORE than the square of the reduction of voltage applied to the motor terminals. So for a loaded motor that has ½ voltage applied to the stator, the value of average accelerating torque will be less than ¼ and the accel time will be more than 4 times that of an across the line start. How much more is greatly dependant on the load torque requirement. Variable torque loads will only slightly change this value whereas constant torque loads may greatly increase this value.

Lastly, for the issue of motor heating, since the majority of motor heating is due to I squared R losses, we can state that the reduction of power dissipated in the motor during soft starting is proportional to the square of the reduction of current flow into the motor. Since we know that the reduction in current flow is proportional to the reduction of motor stator voltage, we can then conclude that the reduction of power dissipation in the motor during a soft start is proportional to the square of the reduction of voltage to the stator. Since energy is the product of power times time, we can state that reduction of energy in the motor is proportional to the reduction of power generated in the motor and proportional to the increase in time that this power exists in the motor. Now we need to break this analysis up into one for the unloaded motor situation and one for the loaded motor situation.

First for the motor with no load torque, we know that power is reduced in proportion to the square of the applied voltage. We also know that the acceleration time is inversely proportional to the square of the reduction of applied voltage. Therefore the amount of energy in the motor is unchanged for an across the line start as compared to a soft start. For example, if the voltage is ½ then current is also ½ power is ¼, accelerating torque is ¼, and accelerating time is 4 times. And since energy is power times time, we have ¼ * 4 = 1 or the same amount of energy.
Next for the motor with a load torque requirement, we know that the value of average accelerating torque is greatly affected by the load torque demand during starting. Therefore, depending on the load torque requirement, the acceleration time is increase by MORE than the square of the reduction in voltage. Therefore, since energy is power times time, the energy dissipated in the motor will be greater, but by how much depends on the load torque requirement. A variable torque load will require very little additional energy whereas a constant torque load may dissipate a substantially larger amount of energy in the motor. Because of this feature, variable torque loads can usually tolerate long acceleration times, whereas, constant torque load acceleration times may only be capable of tolerating soft starting for only a short period of time.

We will now evaluate various methods of soft starting an AC induction motor. We will describe how they function and, using the results of the analysis above, we will define the effect each method has on motor characteristics.

III. AUTOTRANSFORMER STARTING:

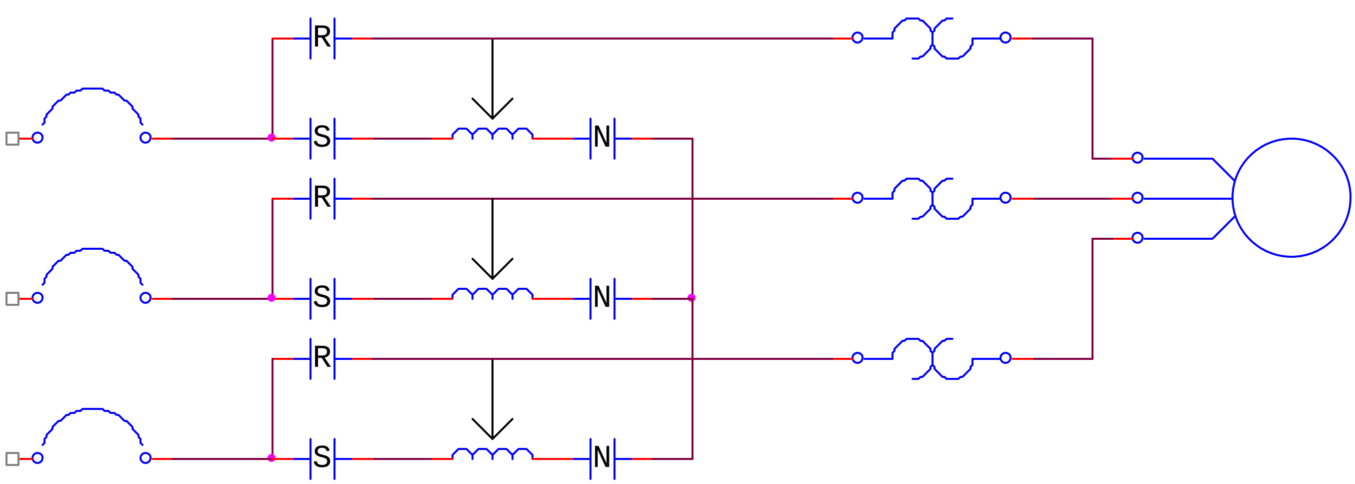

FIGURE #10: Power Circuit of a Single Tap Auto-Transformer Starter:

The power circuit for an autotransformer is shown above in Figure #10. Normally, an autotransformer provides a certain amount of taps to reduce the voltage to the motor terminals, thereby reducing motor current and torque during start. Either timers or centrifugal switches can be used to change tap settings during start. Either an open or closed transition can be provided between settings. Open transition is simpler for logic but can cause current transients during the transition between taps. The closed transition requires more logic but has a smoother transition between tap changes. As we have derived above, as the voltage to the motor winding is reduced, the current to the motor is reduced in direct proportion while the torque value is reduced by the square of the reduction of voltage. A common torque / speed curve for an autotransformer is shown below in Figure #11 and compared to the across the line torque / speed curve. The ATL curve is shown in red and the autotransformer curve is shown in blue.

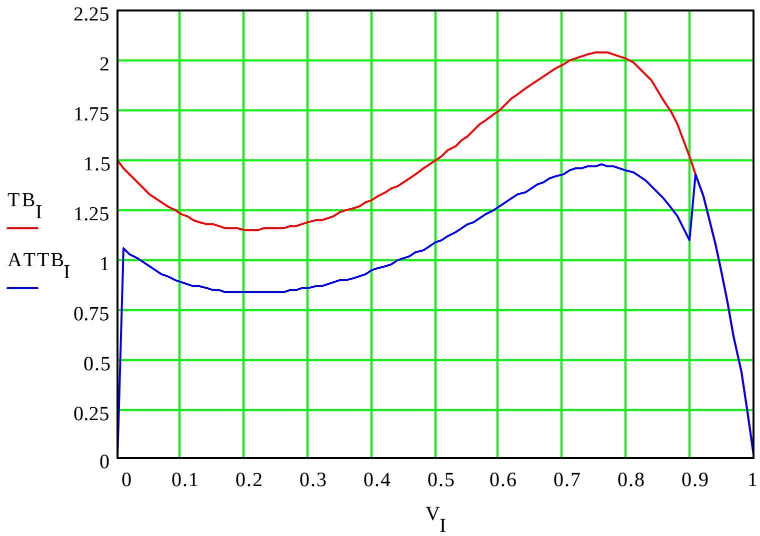

FIGURE #11: Torque / Speed Curve for Autotransformer (85% tap) Starter Compared to ATL Starter:

Some of the advantages of the autotransformer is the reduction in line current as compared to motor current due to transformer action. The drawbacks of the autotransformer are size, cost, limitation of taps, and logic required to implement. They tend to be very large and expensive compared with other solutions. They have a very limited number of taps and, as the number of taps increases, the control logic quickly becomes more difficult.

IV. PRIMARY RESISTOR:

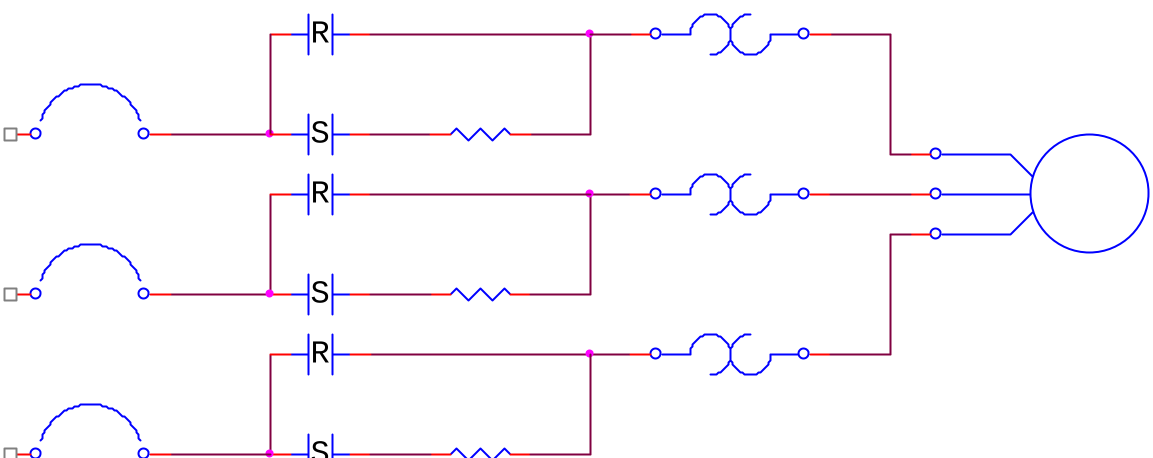

FIGURE #12: Power Circuit of a Primary Resistor Starter:

The power circuit for the primary resistor starter is shown above in Figure #12. The primary resistor starter initially creates a voltage divider by placing high power resistors in series with the motor stator circuit to lower the voltage to the stator of the motor until the run contact is engaged thereby reducing motor current and torque during start. Either timers or centrifugal switches can be used as the signal to shunt out the resistor grid when motor approaches full speed. Either an open or closed transition can be provided between start and run. As we have derived above, as the voltage to the motor winding is reduced, the current to the motor is reduced in direct proportion while the torque value is reduced by the square of the reduction of voltage. A common torque / speed curve for a primary resistor starter is shown below in Figure #13 and compared to the across the line torque / speed curve. The ATL curve is shown in red and the primary resistor starter curve is shown in blue.

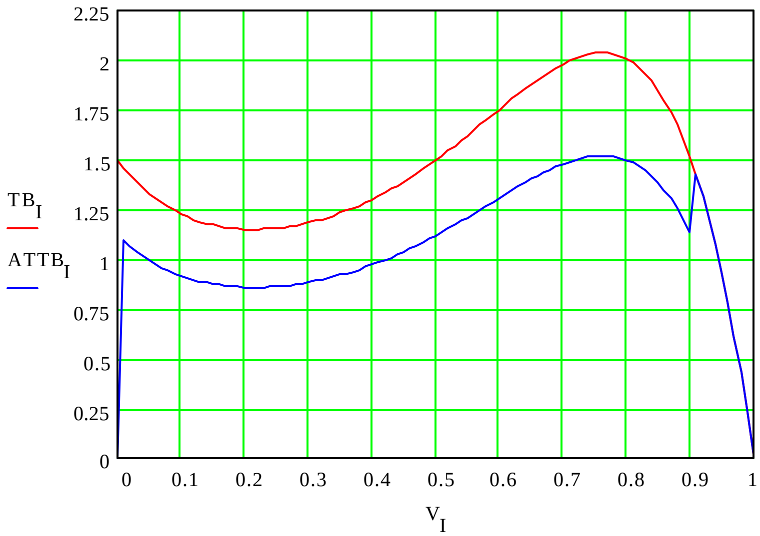

FIGURE #13: Torque / Speed Curve for Primary Resistor Starter Compared to ATL Starter:

One of the advantages of the primary resistor starter is the reduction in line current while maintaining an improved power factor. The drawbacks of the primary resistor starter is additional heat dissipation during start and limitation of initial voltage setting. They tend to be very large due to the heat dissipation requirement and, to change the initial voltage to the stator, the resistance must be changed which is usually a very time consuming and expensive option.

V. PRIMARY REACTOR STARTING:

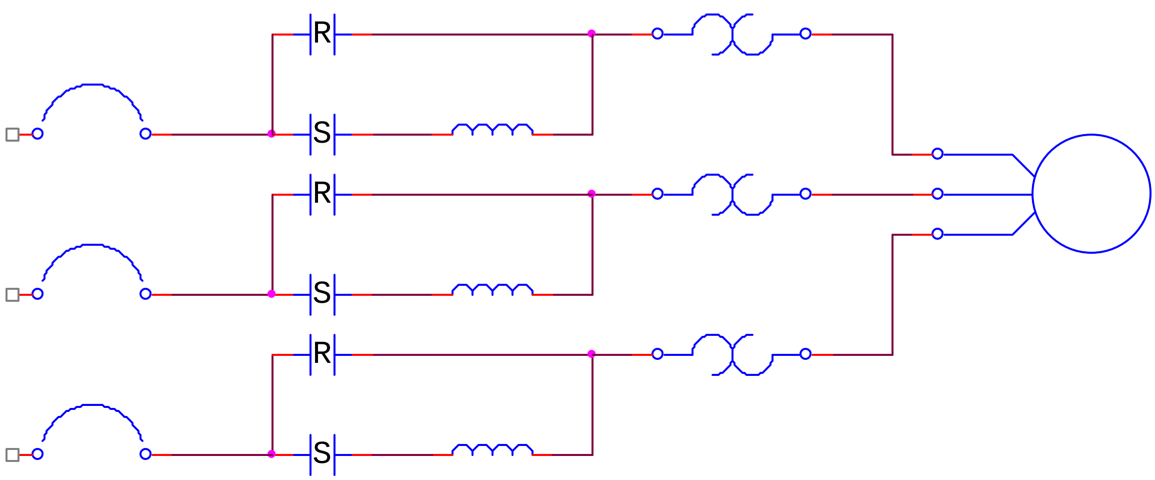

FIGURE #14: Power Circuit of a Primary Reactor Starter:

The power circuit for the primary reactor starter is shown above in Figure #14. The primary reactor starter initially creates a voltage divider by placing reactive elements in series with the motor stator circuit to lower the voltage to the stator of the motor until the run contact is engaged thereby reducing motor current and torque during start. Either timers or centrifugal switches can be used as the signal to shunt out the reactor when motor approaches full speed. Either an open or closed transition can be provided between start and run. As we have derived above, as the voltage to the motor winding is reduced, the current to the motor is reduced in direct proportion while the torque value is reduced by the square of the reduction of voltage. A common torque / speed curve for a primary reactor starter is shown below in Figure #15 and compared to the across the line torque / speed curve. The ATL curve is shown in red and the primary reactor starter curve is shown in blue.

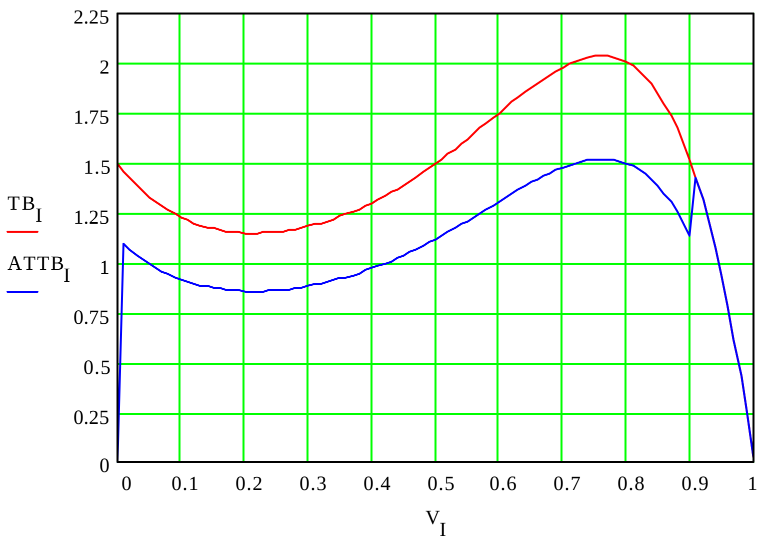

FIGURE #15: Torque / Speed Curve for a Primary Reactor Starter Compared to ATL Starter:

One of the advantages of the primary reactor starter is the reduction in line current with less heat dissipation than the primary resistor circuit. The drawbacks of the primary reactor starter is the poor power factor caused by the additional inductance in the start circuit and the limitation of initial voltage setting.

VI. SERIES/PARRALLEL STARTING:

In series/parallel connection, both ends of each motor winding are provided to allow connection in either series or parallel connection. Therefore, unlike the primary resistor, primary reactor or the autotransformer, this method changes the characteristic motor impedance to reduce current and torque. Initially, the motor is connected in series. The impedance of the motor in this condition is 4 times the impedance seen when the windings are in parallel. Therefore, the current is ¼ of the locked rotor condition. Also, in series connection, the voltage drop across each winding is ½ line voltage so the torque available is ¼ LRT. Once the motor is near full speed, the winding are then reconfigured in parallel. Either timers or centrifugal switches can be used to change motor strapping. A common torque / speed curve for a series / parallel starter is shown below in Figure #16 and compared to the across the line torque / speed curve. The ATL curve is shown in red and the series / parallel starter curve is shown in blue.

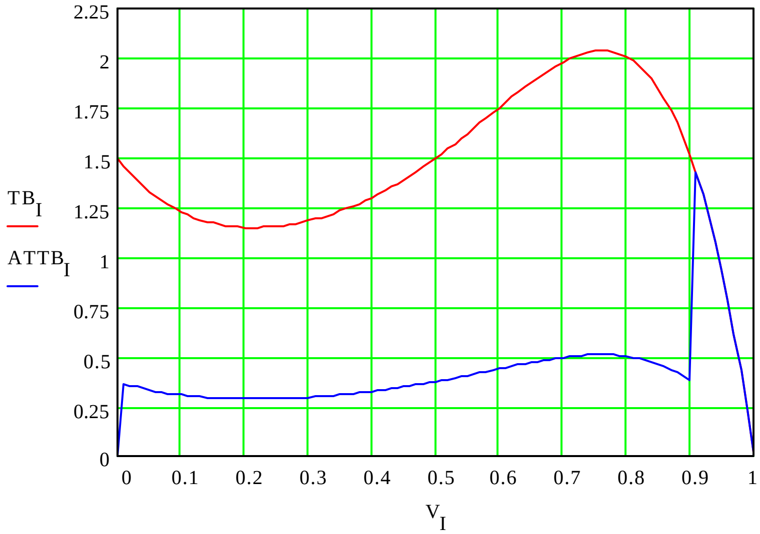

FIGURE #16: Torque / Speed Curve for Series / Parallel Starter Compared to ATL Starter:

Some of the advantages of the Series / Parallel starter is the reduction in line current with respect to torque available. Unlike reduced voltage methods where the current is reduced by ½ when torque is reduced by ¼, the series parallel starter current is reduced by ¼ when torque is reduced by ¼. The drawbacks of the autotransformer are limitation of taps and logic required to implement. They only have two settings and the value of starting voltage cannot be adjusted. Also, the motor must be specially wound which usually adds cost to the motor.

VII. WYE/DELTA STARTING:

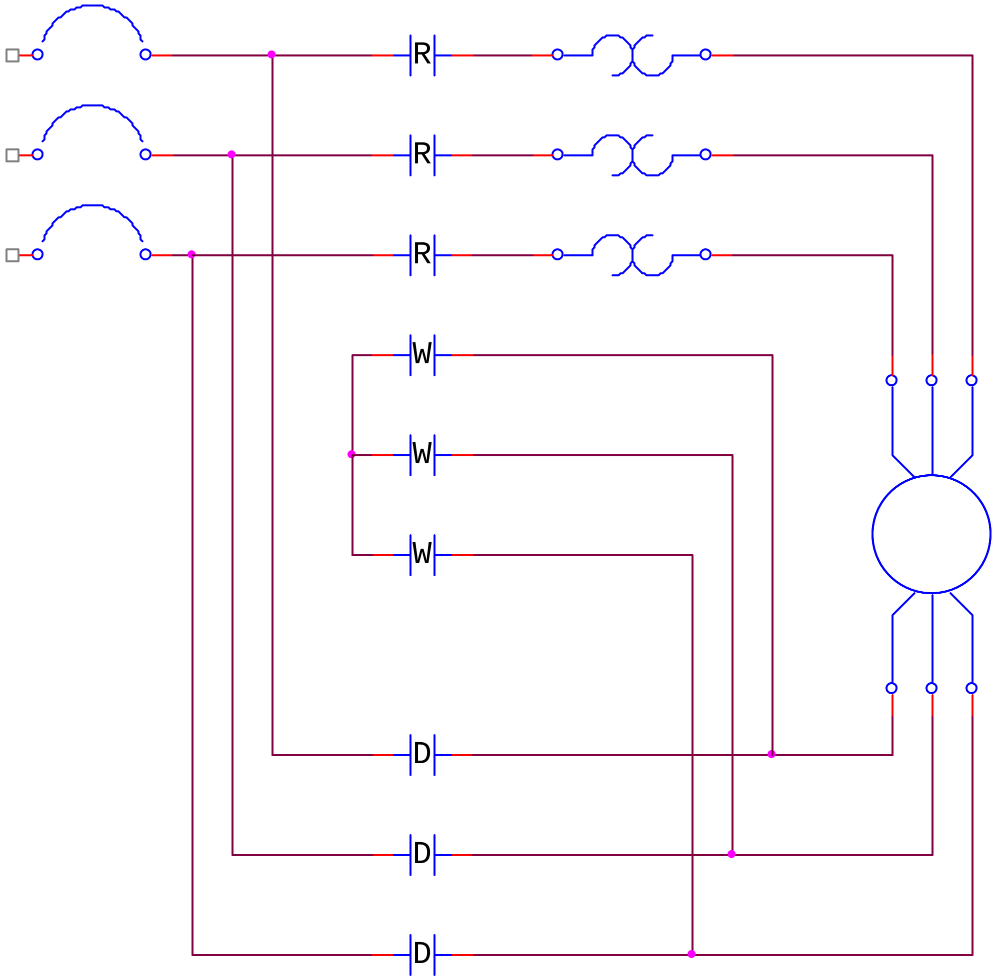

FIGURE #17: Power Circuit of a Wye / Delta Starter:

In the wye / delta starter, both ends of each motor winding are provided to allow connection in either wye or delta connection. Therefore, unlike the primary resistor, primary reactor or the autotransformer, this method changes the characteristic motor impedance similar to the series / parallel starter except the configuration is different. Initially, the motor is connected in wye. The impedance of the motor in this condition is 3 times the impedance seen when the windings are in delta. Therefore, the current is 1/3 of the locked rotor condition. Also, in the wye connection, the voltage drop across each winding is 57.7% line voltage so the torque available is 33.3% LRT. Once the motor is near full speed, the winding are then reconfigured in delta. Either timers or centrifugal switches can be used to change motor strapping. A common torque / speed curve for a wye / delta starter is shown below in Figure #18 and compared to the across the line torque / speed curve. The ATL curve is shown in red and the wye / delta starter curve is shown in blue.

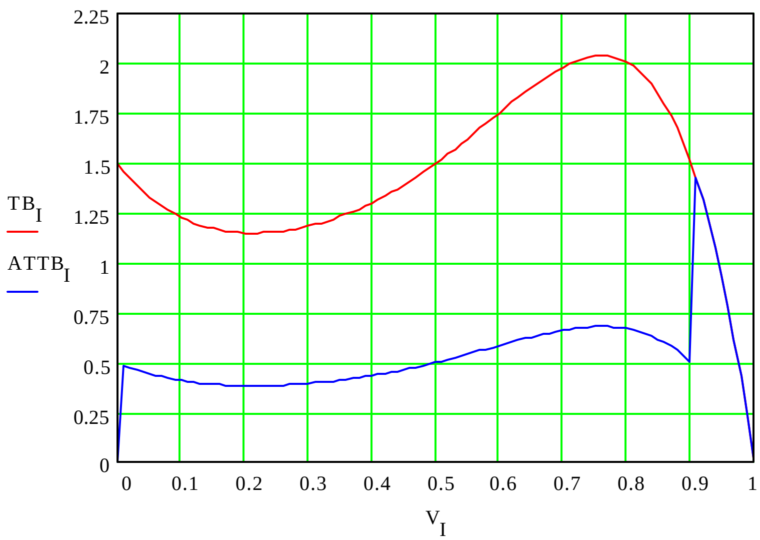

FIGURE #18: Torque / Speed Curve for Wye / Delta Starter Compared to ATL Starter:

Some of the advantages of the wye / delta starter is the reduction in line current with respect to torque available. Unlike reduced voltage methods where the current is reduced by 57.7% when torque is reduced by 1/3, the wye / delta starter current is reduced by 1/3 when torque is reduced by 1/3. The drawbacks of the wye / delta starter is the fact that the motor must be specially wound to provide both ends of each winding. This adds cost to the motor. Also, the starting voltage is fixed at 57.7% of the line-to-line voltage. Also, three contactors must be used and two must be mechanically interlocked to prevent the possibility of a line to line short should the contacts malfunction.

VIII. PART WINDING STARTING:

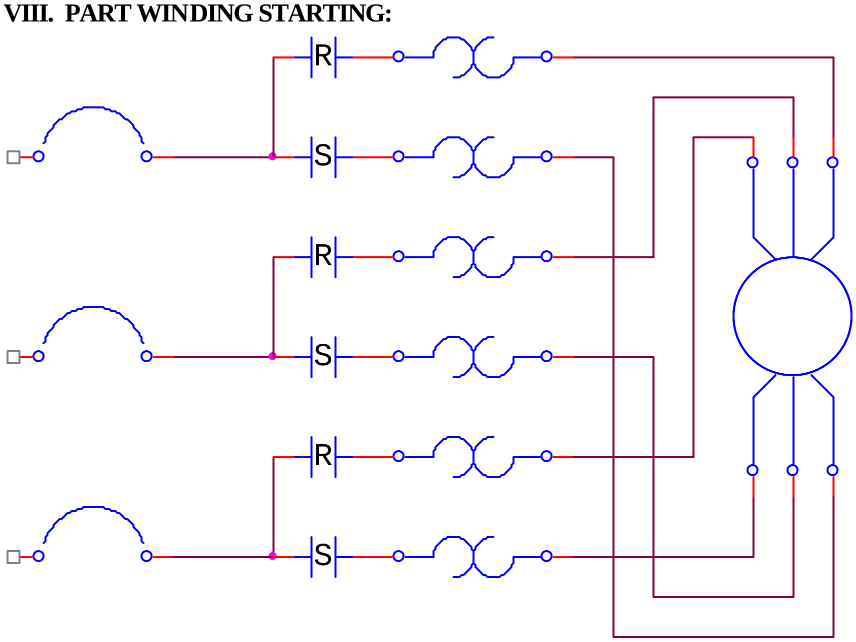

FIGURE #19: Power Circuit of a Part-Winding Starter:

The power circuit for a part-winding starter is shown above in Figure #19. Like the series / parallel and the wye / delta starters, the part winding starter does not reduce the voltage to the motor but reconfigures the windings to provide higher impedance during start. Initially, only one set of windings is energized providing twice the impedance that the motor would see in the run mode. Since the impedance is 2 times the running impedance, the current is 50% of the LRA value. Also, the torque is 50% of the LRT. Once the motor is near full speed, the second set of windings is energized. A common torque / speed curve for a part winding starter is shown below in Figure #20 and compared to the across the line torque / speed curve. The ATL curve is shown in red and the part winding starter curve is shown in blue.

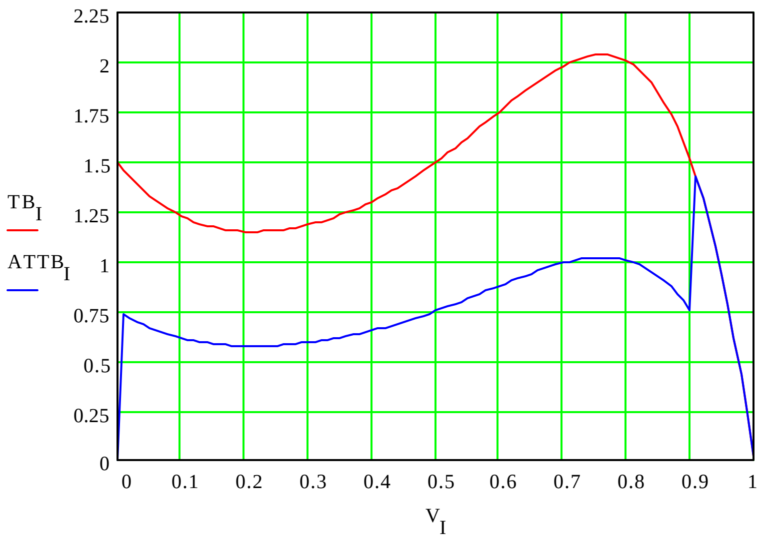

FIGURE #20: Torque / Speed Curve for Part Winding Starter Compared to ATL Starter:

Some of the advantages of the part-winding starter is the reduction in line current with respect to torque available. Unlike reduced voltage methods where the current is reduced by 70.7% when torque is reduced by 1/2, the part winding starter current is reduced by 1/2 when torque is reduced by 1/2. The drawbacks of the part winding starter is the fact that the motor must be specially wound to provide both 2 sets of windings each to be brought out for connection. This adds cost to the motor. Also, the there are only two settings of motor impedance, which provides no adjustment for starting torque variations.

IX. SOLID STATE SOFTSTART STARTING:

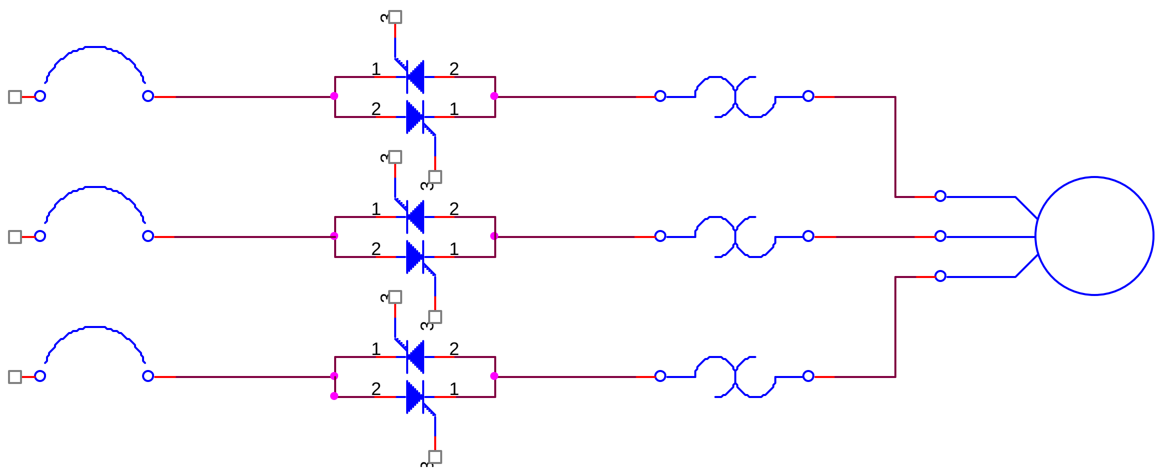

FIGURE #21: Power Circuit of a Solid-State Soft Start Starter:

The power circuit for a solid state soft start unit is shown above in Figure #21. The solid-state soft start unit is based upon the SCR (Silicon Controlled Rectifier), which is placed in series with the motor stator windings. When the start command is given, the SCRs fire at some phase angle that is dictated by the programmable settings in the solid-state control circuitry. Normally, some value of current is chosen as a limit point and the SCRs are gated at the correct angle of conduction in order to maintain current at this set point throughout the start curve. Once the motor is at full speed, the SCRs smoothly reach full conduction angle. At this point only the P-N-P-N silicon voltage drop is seen across the SCR. The rest of the system voltage is available to the motor. At full speed, this P-N-P-N voltage drop (normally about 1 vac) does cause some heat generation inside the enclosure and, thereby, a very small reduction of system efficiency during running. In cases where this additional heat is not tolerable, a shunt bypass contactor can be used to shunt the current around the SCRs once the motor obtains full speed as see in Figure #22 below:

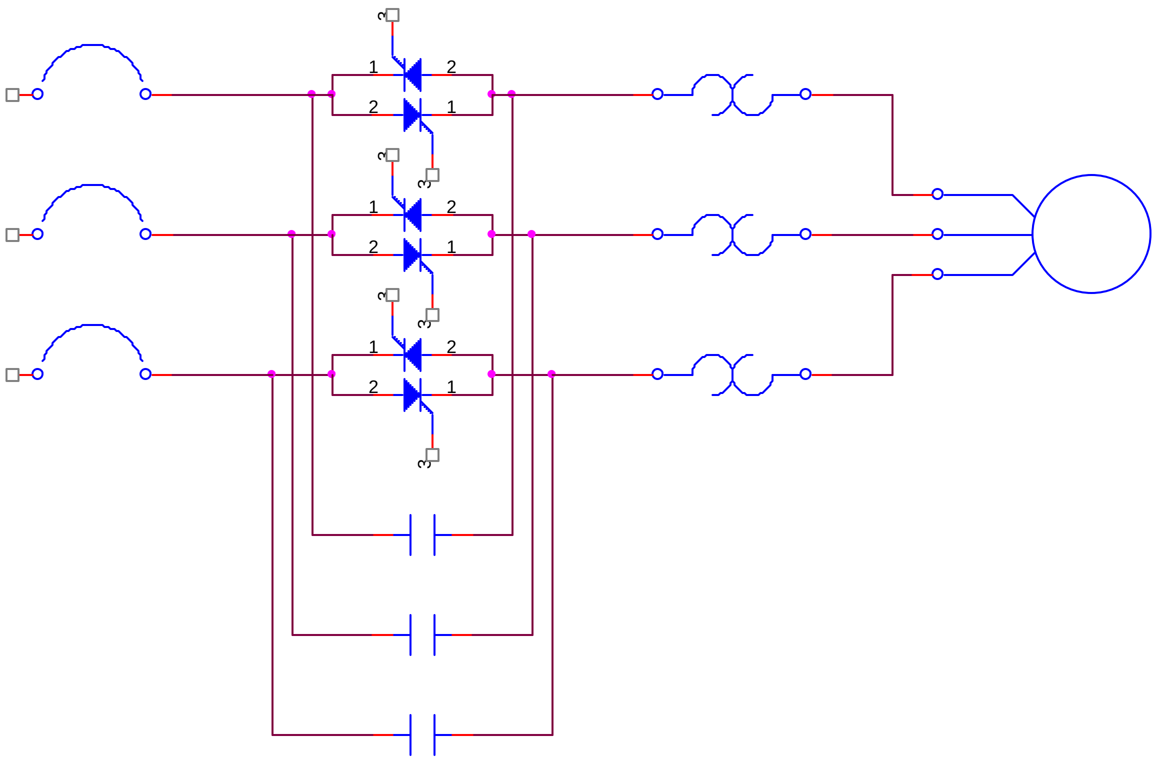

FIGURE #22: Power Circuit of a Solid State Soft Start Starter with Shunt Bypass:

In order to understand how the solid-state device controls the value of voltage to the motor, it is helpful to show the waveform that the motor sees during its start. This can be seen in Figure 23 below.

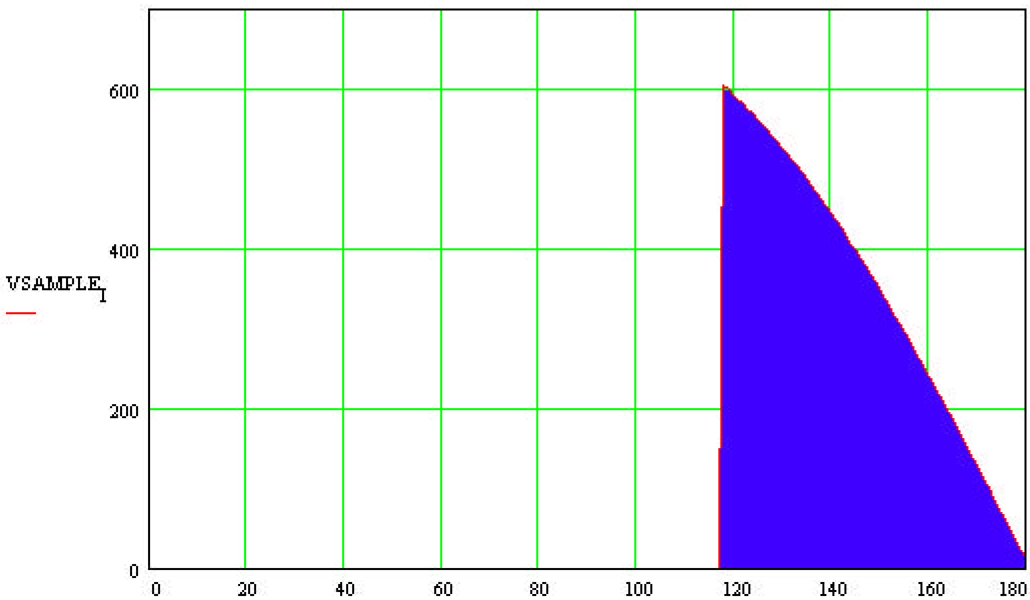

FIGURE #23: Waveform of Voltage at Motor Stator Terminal During Solid State Starting (delay angle = 120 degrees):

This shows that of the total input sine wave, during the first 120 degrees, the SCR is not gated and all the input voltage is dropped across the SCR. This leaves no voltage for the motor. At 120 degrees the SCR is gated and passes system voltage to the motor for the rest of the sine wave. At the current zero cross the SCR naturally commutates off. Since the value of voltage to the motor is the area under the voltage waveform at the motor (the area in blue in figure 20), it is obvious that, as the angle of delay is increased, the value of motor voltage is decreased. Since there is no limitation to the angles of delay that can be chosen, There is an infinite amount of settings for output voltage between 0 and line voltage that can be delivered to a motor to obtain the current limit set point chosen. This is unlike any of the other electro-mechanical methods of soft starting described above. Those all have some well defined transition point that causes a torque surge in the load. This torque surge does not exist in a solid-state soft start since the transitions are infinitely small. A common torque / speed curve for a solid-state soft start unit is shown below in Figure #24 and compared to the across the line torque / speed curve. The ATL curve is shown in red and the solid-state soft start curve is shown in blue. Note the lack of any sudden torque surge that exist in the electro-mechanical soft start methods listed above.

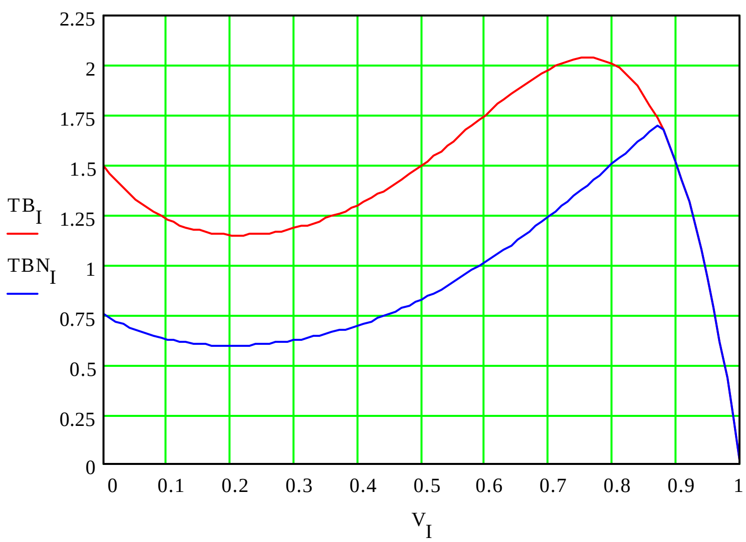

FIGURE #24: Torque / Speed Curve for Solid State Soft Starter (current limited to 3 times FLA) Compared to ATL Starter:

One drawback of the solid-state soft start is that, unlike the other electro-mechanical methods, the waveform during starting, is not a pure sine wave. The leading edge of the waveform contains some harmonics. However, these harmonics only exist for a short time during starting since, once the motor is at speed, the output waveform is a complete sine wave. The advantages of a solid state soft start are many. The lack of a torque spike for transition is one. The ability of nearly infinite values of starting torque is another. The ability to set the peak current draw is another. The lack of any mechanical contactors is another. This is because, with mechanical contactors, there is both wear on the moving parts as well as arcing every time the contactor is opened. The solid-state device has no moving parts and no arcing occurring during transition so the maintenance of the solid-state device is very much reduced. In addition to these features of the power devices, since the solid-state controller is driven by solid-state electronics instead of electro-mechanical relay logic, many additionally features commonly come with solid-state control. Normally, there is no separate mechanical overload relay required since motor protection can be achieved digitally. With the digital controller a soft stop feature can be achieved. This is used in pumping applications where, if power is removed immediately during a stop command, water hammer can cause damage to the mechanical system. With the soft stop feature, once the stop command is given, the voltage is smoothly lowered to the motor to slowly reduce the motor torque and therefore system pressure thus eliminating water hammer. With the digital controller, a DC injection brake can be achieved. This is done in high system inertia applications to reduce the stop time of the load. When a variable amount of DC current is injected to the motor after a stop command is given, the rotor tries to line up with the DC field created inside the stator. This causes the motor to stop more quickly in high system inertia applications than just removing ac voltage from the stator of the motor. With the digital controller, a feature called electronic shear pin can be achieved. This feature monitors the torque delivered to the system by the motor and, if the torque level exceeds the maximum tolerable level of the system, the controller will remove power from the motor thus protecting the mechanical system from excessive torque levels. With the digital controller, a feature called short circuit trip can be utilized. In most motor applications, the maximum current the motor could ever pull from the system is the LRA (locked rotor amperage) of the motor which is normally 6 to 8 times current. The short circuit trip feature can sense current levels

above this indicating that an abnormal magnitude of current is flowing and remove power from the motor usually in about 10 mS. Similarly there is a feature called shunt trip on most soft starts. This feature looks for current flow with no stop command and, if current flow were detected, then a signal would be sent to trip an external breaker removing power from the unit. This is done to protect the motor from a failed device in the soft start. It would be akin to having protection on a motor starter to protect the motor in the event one or more poles of the starter weld shut. In addition to these basic features, some solid-state devices can accept RTD (resistive thermal devices) that are imbedded in the motor to provide true motor protection.
As the above paragraph shows, solid-state motor control offers many beneficial features other than soft starting, that the traditional electro-mechanical devices cannot. One last method of soft starting an AC induction motor is with the use of the variable frequency drive (VFD).

X. VARIABLE FREQUENCY DRIVE STARTING:

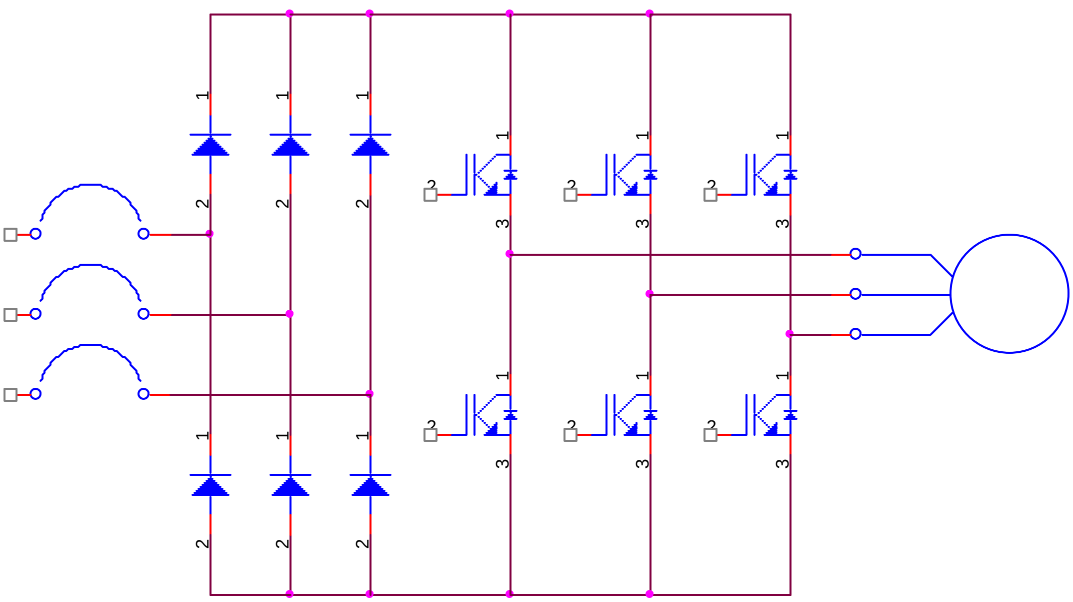

FIGURE #25: Power Circuit of a Variable Frequency Drive:

Although there are many different topologies, a generic power circuit for a variable frequency drive (VFD) is shown above in Figure #25. The VFD takes the line power and rectifies it to a DC value and then inverts it back to an AC voltage output of variable frequency and voltage. One of the methods used is called pulse width modulation (PWM). PWM means that each pulse width is modulated or varied to simulate the desired output voltage magnitude and frequency. An example of what the PWM waveform looks like appears in Figure #26 below.

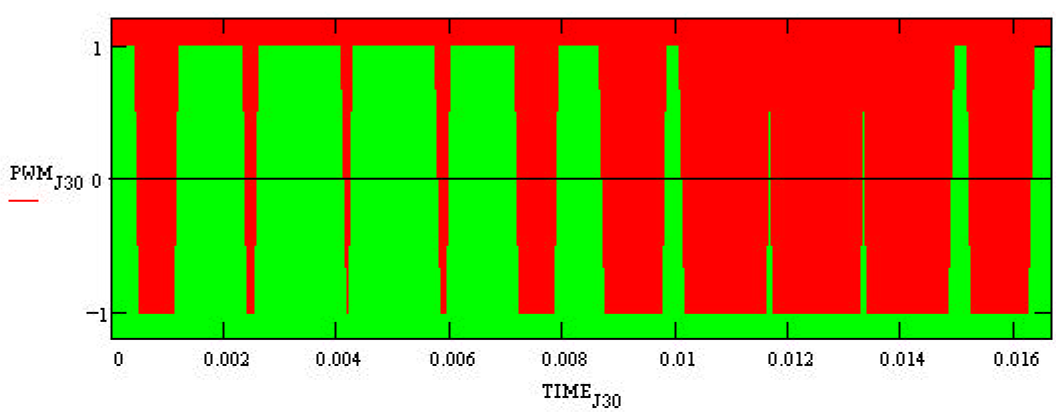

FIGURE #26: Sample Output Voltage Waveform of a Variable Frequency Drive:

Since the output of the VFD can have a variable voltage and variable frequency, the VFD can provide full load torque throughout the speed range of 0 rpm to rated rpm of the motor. If you will recall the equation for motor torque developed earlier in this paper, the torque is a function of both stator voltage and the resistance / impedance internal to the motor. The equation for torque is:

$$T := \frac{(|Van|)^2}{\Omega s} \cdot \left[\frac{\frac{a^2 \cdot Rr}{S}}{\left(Rs + \frac{a^2 \cdot Rr}{S} \right)^2 + Xe^2} \right]$$

Initially, let us make the assumption that resistance of the motor is much less than the reactance. If Xe is much larger than Rs or Rr, then the term on the right varies inversely with frequency and we can make the following statement:

$$T := \frac{(|Van|)^2}{\Omega s^2} \cdot K$$

Or, rewritten one more time to see the proportionality between torque, voltage, and frequency:

$$T := \left(\frac{|Van|}{\Omega s} \right)^2 \cdot K$$

Therefore, ideally, the motor available torque is proportional to the square of the ratio of stator applied voltage to the stator applied frequency. Therefore, to provide full load motor torque throughout the speed range of 0 rpm to full load RPM, the voltage output of the VFD normalized to the motor nameplate voltage must stay in proportion to the output frequency normalized to the rated motor frequency. This is known as a constant torque volts to hertz (V/F) pattern. In practicality, even though Rs and Rr are much less than Xe, they are not zero and, as such, when the applied stator frequency is reduced the applied voltage is reduced but not by as much as the reduction in stator frequency. This is because, when the stator frequency is reduced, the value of the denominator on the right is reduced but not by the same scale as the reduction in frequency due to the constant values of Rr and Rs. Because of this, the denominator of the full equation is reduced but by an amount slightly less than the magnitude of the reduction of frequency. To maintain full torque, the voltage must be reduced in proportion to the denominator. Since the reduction of the denominator is slightly less than the reduction of applied frequency, then the reduction of the voltage normalized to motor nameplate voltage is slightly less than the reduction of applied frequency normalized to the motor nameplate frequency. For example, in an ideal application when starting a 60Hz, 480 VAC motor at an initial frequency of 1 Hz, the ideal value for motor voltage at 1Hz would be (1/60)*480 = 8VAC. However, due to the effect of the non proportionality listed above, it is more common to provide about 30 VAC at 1 Hz to ensure full torque capability of the motor. Now if you refer back to the torque / speed curve chart you will see that, at full torque the motor draws only full voltage. Since the VFD is applying a variable frequency, it is maintaining the motor at or above full load speed at all times and, therefore, it maintains motor current draw at or below motor FLA at all times (assuming sufficient time is allows for acceleration). This capability of the VFD to maintain full load torque throughout the speed curve while maintaining motor current at or below FLA makes the VFD an excellent method of reduced voltage starting.

Because of the infinite amounts of combinations of applied voltages and applied frequencies that are possible in a VFD, a torque speed curve is very difficult to describe. However, a common technique for looking at the torque speed curve for a motor driven by a VFD is to plot the motor toque / speed curve at several applied frequencies, assuming a constant torque voltage pattern is used, and then plot these motor torque / speed curves on one graph. A torque / speed curve using increments of 6Hz for the applied frequencies is shown below in Figure #27 for a standard NEMA B motor.

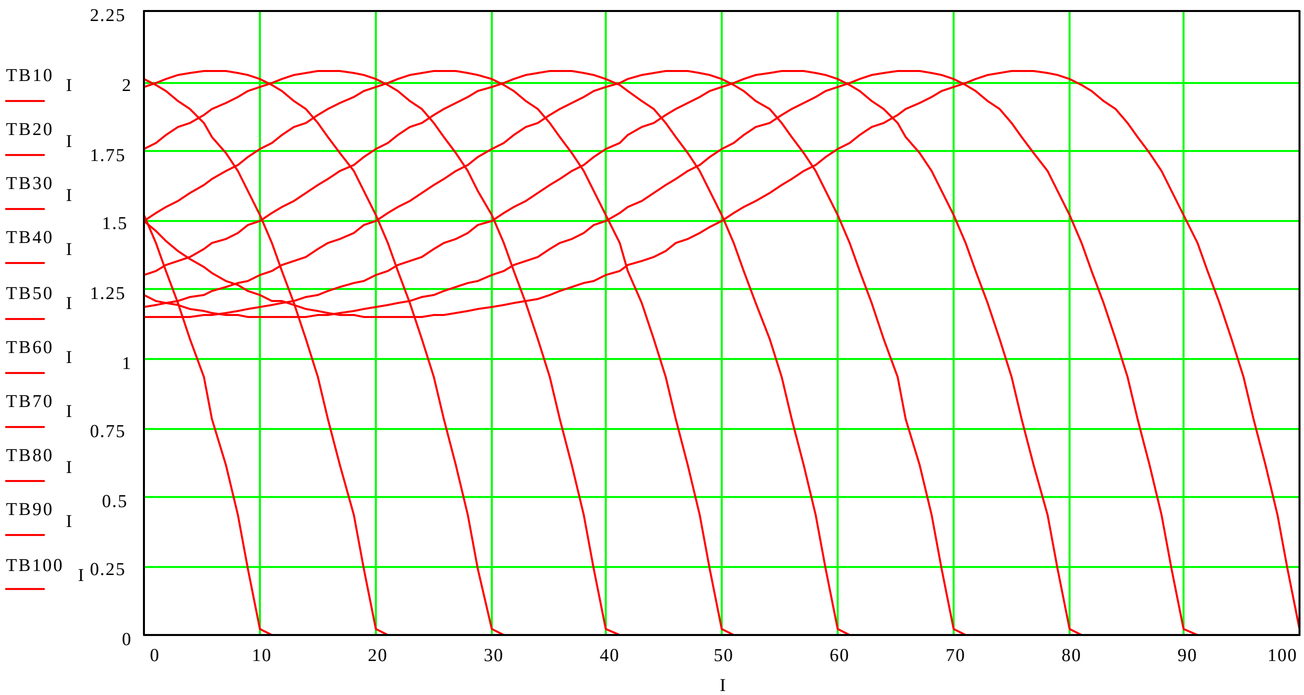

FIGURE #27: Torque / Speed Curve for a VFD Set to 10% (6Hz) Increments:

Notice that even though the applied frequency to the motor changes, the rated slip of the motor (i.e., the difference in RPM between synchronous and full load speeds) does not change. This is an important issue to understand. Since a motor rotor needs to be able to slip a certain amount of RPM to provide torque, the stator rotational field must be rotating at a minimum of that slip speed for full load torque to be delivered. For example, for a 5% slip, 60 Hz motor, for the motor to provide full load torque, the frequency applied to the stator must be, at minimum, 5%*60 = 3Hz. Realize, that if 3 Hz is applied to this motor and full load torque is demanded from the shaft of the motor the motor will deliver this torque but the shaft will not be turning. That is because, the applied frequency equals the rated slip of the motor. This is why to maintain speed regulation while also allowing a motor to deliver full load torque, slip compensation is utilized. This feature basically detects the motor slip and then increases automatically the applied frequency to pull the motor to the desired speed.

One drawback of the variable frequency drive is that, unlike the other electro-mechanical methods, the voltage waveform on the output is not a pure sine wave and the current waveform on the input is also not a pure sine wave. Therefore, current harmonics exist on the input of the VFD and voltage harmonics exist on the output of the VFD. If harmonics are an issue for the application, then filtering can be applied to the input / output of the drive to filter the high frequency harmonics. For more information on this issue, please refer to an article I wrote entitled **SEMICONDUCTOR GENERATED WAVEFORM MODELING And Waveform Harmonic Reduction by Filter Modeling.** Another possible disadvantage for the VFD is cost. In larger power and voltage ratings VFDs are several times the cost of any of the other methods of soft starting. Recently, however, the cost of small power, 480 volt and less VFDs have been reduced to levels comparable with other methods of control.

The advantages of a VFD are many. The lack of a torque spike during transition is one. The ability of infinite values of starting torque is another. The ability to set the peak current draw and peak torque value is another. The lack of any mechanical contactors is another. With mechanical contactors, there is both wear on the moving parts as well as arcing every time the contactor is opened. The VFD has no moving parts and no arcing occurring during transition like electro-mechanical soft start units. Therefore, the maintenance of VFD is very much reduced. In addition to these features of the power devices, since the VFD is driven by solid state electronics instead of electro-mechanical relay logic, many additionally features commonly come with VFD control. Normally, there is no separate mechanical overload relay required since motor protection can be achieved digitally. With the VFD the deceleration rate is also controlled. This is used in pumping applications where, if power is removed immediately during a stop command, water hammer can cause damage to the mechanical system. With the VFD, once the stop command is given, the voltage and frequency are smoothly lowered to the motor to slowly reduce the motor speed and torque and therefore system pressure thus eliminating water hammer. With the VFD a dynamic brake function can be achieved. This is done in high system inertia applications to reduce the stop time of the load. With the VFD, a feature called over torque protection can be achieved. This feature monitors the torque delivered to the system by the motor and, if the torque level exceeds the maximum tolerable level of the system, the controller will reduce the torque to the system, thus protecting the mechanical system from excessive torque levels. With the VFD, a feature called short circuit trip is utilized. Since motor current normally never exceeds FLA of the motor, the threshold is usually 2 times motor FLA. If the VFD senses current at 2 times FLA, then the drive will remove power from the motor usually in about 10 uS. In addition to these basic features, some VFDs can accept RTD (resistive thermal devices) that are imbedded in the motor to provide true motor protection. The VFD can also control the speed of the motor at speeds other than nominal. The VFD can also be used to convert single phase to 3 phase power as well as voltage conversion from 120 single phase to three phase 240. The VFD can also perform solid state reversing of the motor without the use of external reversing contactors. There are many other features available when using VFD control of a motor.

XI CONCLUSION:

We have described basic AC induction motor theory. We have derived equations to describe various motor electrical and mechanical properties. Next we discussed motor across the line characteristics and then we determined motor characteristics during soft starting. Lastly, we have reviewed several of the more common methods of soft starting motors and evaluated their respective advantages and disadvantages.

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