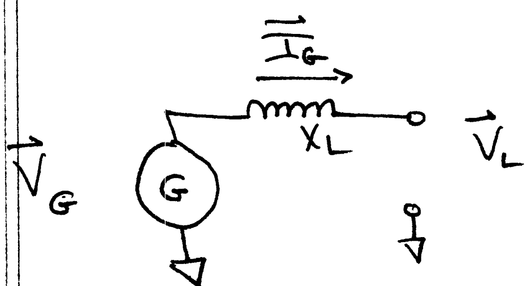
Derivation of Power Transfer function (single line eqv. ckt)

Given the simple model below of a generator and a transmission system with some inductance between the two, we can derive fundamental equations for Real Power and Reactive Power flow between the generator & system.



Lets use Generator Voltage as our Reference Vector so:

$$\vec{V}_G = |V_G| \angle 0^\circ$$

The Line Voltage is at some other magnitude & displacement so:

$$\vec{V}_L = |V_L| \angle \delta$$

Now we will Mathematically manipulate these two equations to find out what controls Real and Reactive Power flow out of generator.

$$\vec{I}_G = \frac{\vec{V}_G - \vec{V}_L}{jX_L} = \frac{|V_G| - |V_L| \angle \delta}{jX_L} = \frac{|V_G|}{jX_L} - \frac{|V_L|[\cos\delta + j\sin\delta]}{jX_L}$$

$$\vec{I}_G = \frac{|V_G|}{jX_L} - \frac{|V_L|\cos\delta}{jX_L} - \frac{j|V_L|\sin\delta}{jX_L} = -j\frac{|V_G|}{X_L} + j\frac{|V_L|\cos\delta}{X_L} - \frac{|V_L|\sin\delta}{X_L}$$

$$\vec{I}_G = \frac{|V_L| \sin\delta}{X_L} + j\left[\frac{|V_L| \cos\delta}{X_L} - \frac{|V_G|}{X_L}\right]$$

$$\vec{I}_G^{*} = \frac{|V_L| \sin\delta}{X_L} + j\left[\frac{|V_G|}{X_L} - \frac{|V_L| \cos\delta}{X_L}\right]$$

$$\vec{S}_G = \vec{V}_G \cdot \vec{I}_G^{*} = |V_G| \cdot \left[\frac{|V_L| \sin\delta}{X_L} + j\,\frac{|V_G|}{X_L} - \frac{|V_L| \cos\delta}{X_L}\right]$$

$$\vec{S}_G = \left(\frac{|V_G||V_L| \sin\delta}{X_L}\right) + j\left(\frac{|V_G||V_G|}{X_L} - \frac{|V_G||V_L| \cos\delta}{X_L}\right)$$

$$\vec{S}_G = \left(\frac{V_G V_L \sin\delta}{X_L}\right) + j\left(\frac{V_G^2 - V_G V_L \cos\delta}{X_L}\right)$$

$$\vec{S}_G = P + jQ \qquad \text{Therefore}$$

$$P = \left(\frac{V_G V_L \sin\delta}{X_L}\right), \qquad Q = \left(\frac{V_G^2 - V_G V_L \cos\delta}{X_L}\right)$$

Note, if $\delta = 0$, There is No Real Power transmitted. For a given Magnitude of V_G & V_L, as we increase the Value of δ, we increase Real Power transfer. Also Note Point of Maximum Real Power transfer is at $\delta = 90^\circ$. Any values Larger Than 90°, Result in Reduced Real Power xfer and will Lead to The generator slipping a Pole. It is for this Reason that δ is known as the Power angle and δ is Controlled to Be Between 0° and 90° (actually smaller window, but this is Later in This Paper) ***

If δ Becomes a negative value (ie. Generator voltage LAGS System Voltage Angle) Then Power Is delivered To The generator and The generator Begins To motor.

NOTE ALSO That if $|V_G| = |V_L|(\cos\delta)$, There is No Reactive Power Transfer Regardless of The Value of δ. as The Magnitude of V_G is Raised above The Magnitude of $V_L(\cos\delta)$, Reactive Power IS transferred To The system. also NOTE if The Magnitude of V_G is Reduced Below The Magnitude of V_L, Reactive Power is Absorbed By The generator from the system.

Because Of The Relationship Between δ and Real Power Transfer, δ is KNOWN as The "POWER" Angle. Real Power transfer is Controlled By Controlling The Phase Angle Between Generator Voltage and System Voltage. δ is Controlled at The generator by Controlling The governor Valves To The Turbine. Because of The Relationship Between The Magnitude of generator and Line Voltage, Reactive Power Transfer is Controlled By Controlling The Magnitude of generated Voltage. Generator Voltage is Controlled By adjusting generator field Current,

** Maximum Power angle δ_{cr} is dependent on max system fault clearing Time. This Angle is Maintained Low enough so that, during a system fault, when The Power angle Advances, it does not exceed The maximum angle δ_{max} where Pole Slip will occur

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